Control of a Slow-Moving Space Crane as an Adaptive Structure

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If we assume that the space crane is a statically determinate truss with length-adjustable bars and take as controls the length adjustments of these bars, the computation of the incremental controls corresponding to the motion of a payload along a trajectory is given in terms of the inverse-transpose of matrix B of the joint equilibrium equations Bs = p, where s lists the bar forces and p the nodal loads. An algorithm with $\mathcal{O}(n^2)$ computational complexity and $\mathcal{O}(n)$ storage demand is used for obtaining the inverse of the nth-order sparse matrix B. The compensation of the controls for elastic deformations and support movements is shown. The crane is assumed to be moving sufficiently slowly so that no vibratory motion is created during its maneuver. To simplify the computations, a zero-acceleration field is assumed in the workspace of the space crane. It is shown that the computations may be done automatically and in real time by an attached processor once the characteristics of the crane's maneuver are keyed in.

I. Terminology

THROUGHOUT this paper, an italic boldface letter indicates a free vector, while a lowercase boldface letter denotes a column matrix. Thus, $\boldsymbol{r}$ is a free vector and $\mathbf{r}$ a column matrix. A scalar is represented by an italic lightface letter and a matrix with more than one column by an upper case boldface letter. Thus, t and N are scalar variables, while $\mathbf{B}$ is a matrix with more than one column. Superscripts T and -1 stand for matrix transposition and inversion, respectively. A superscript $'$ to any vector implies that it is derived from a larger vector by deleting a few rows. Thus, $\boldsymbol{v}'(t)$ stands for a vector that is obtained by deleting some of the rows of the vector $\mathbf{v}$. Similarly, a superscript $''$ to a matrix indicates that it is derived from a larger matrix by deleting a few columns. Thus, $[\mathbf{B}^{-1}]''$ and $[\mathbf{B}^{-T}]''$ are matrices obtained from $[\mathbf{B}^{-1}]$ and $[\mathbf{B}^{-T}]$, respectively, by deleting some columns.

II. Introduction

Future space missions such as a manned mission to Mars or establishing a lunar outpost will require the construction of large vehicles and their components in space, as they are too massive to be lifted economically and safely by a heavy launch vehicle or Space Shuttle. Many large-antenna concepts[1-3,6,9] and space station concepts[3] suggested until now employ trusses that can be assembled in space from small modular units. The space construction activity will require large space cranes to move large masses from one location to another. One of the space cranes being considered by NASA is a truss structure with some members of the truss being length-adjustable. Recently there has been active research to find suitable length-control mechanisms. Some researchers have studied piezo-electric-type actuators[4] and others have considered heaters[7,8] as possible length actuators. Such control mechanisms may be very effective for shape-control applications in which only small length changes in the members are required. For large motion control of a space crane, other mechanisms such as hydraulic actuators that provide large length changes in the elements can be used. Whatever the mechanism finally chosen for the length-controlled members of the space crane, efficient control algorithms for the maneuver of moving a payload from one point to another are needed. Efficient control necessitates that lesser control energy be spent during the maneuver.

Space cranes used in space construction can be adaptive structures.[10] An adaptive structure can change its geometry without creating internal stresses. It is, in general, a statically determinate structure, in particular, a statically determinate truss. The response computation of statically determinate trusses is reviewed in the next section where an algorithm with $\mathcal{O}(n^2)$ computational complexity and $\mathcal{O}(n)$ storage demand is referred to for the required nth-order matrix inversion.

A space crane using the adaptive structure concept is shown in Fig. 1. It is clear from the figure that the crane is essentially a statically determinate truss with some length-control mechanisms installed in some of the members. The end-effector of the crane traces the desired trajectory of the payload upon the activation of the length-control mechanisms to induce appropriate elongations/shortenings in appropriate bars. Because of the adaptive nature of the truss, no stress in the truss members develops when the payload is coasting along its trajectory. The knowledge of the payload's trajectory enables one to compute the corresponding elongations/shortenings of the controlled bars. The minimum-energy trajectory of the payload in the workspace of the space crane and its velocity profile along this trajectory are studied in Sec. 4, where we assume a zero force field in the workspace.

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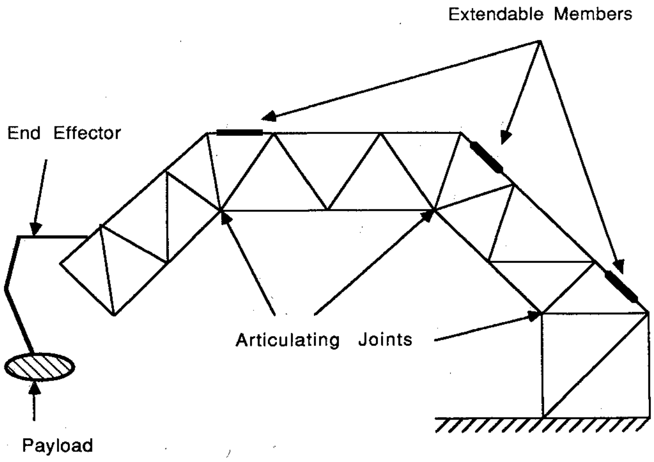

Fig. 1 Space crane as an adaptive structure.

To prevent vibrations, the space crane may be moved slowly such that the dynamic forces on the payload and the truss are sufficiently small, and therefore, the dynamic response of the crane can be ignored. With such a slow-motion assumption, the excitation-response relations summarized in the next section may be used to determine the control variables, i.e., the elongations/shortenings of the controlled bars, at each instant t of the maneuver. Let the q-tuple $\mathbf{v}'(t)$ list the control variables. As described in this work, the computation of $\mathbf{v}'(t)$ may be achieved incrementally. Let $\Delta\mathbf{v}'$ list the incremental controls at time t. It consists of three parts: $\Delta\mathbf{v}'^r$, $\Delta\mathbf{v}'^e$, and $\Delta\mathbf{v}'^s$, such that $\Delta\mathbf{v}' = \Delta\mathbf{v}'^r + \Delta\mathbf{v}'^e + \Delta\mathbf{v}'^s$. Here, $\Delta\mathbf{v}'^r$ lists the incremental controls when the members of the crane are assumed rigid, and the crane has no support movement, $\Delta\mathbf{v}'^e$ lists the corrections compensating for the errors caused by the elastic deformation of the crane due to the interacting force between the payload and the end-effector, and $\Delta\mathbf{v}'^s$ lists the corrections compensating for the deviations from the payload's trajectory due to the crane's support movements. The computations of $\Delta\mathbf{v}'^r$, $\Delta\mathbf{v}'^e$, and $\Delta\mathbf{v}'^s$ are discussed in Sec. V, VI, and VII, respectively.

In Sec. VIII the computational complexity of the computations is given in terms of the characteristic parameters of the crane maneuver. It is shown that the computation of the controls can be performed in real time automatically by an attached processor once the characteristic parameters of the maneuver are keyed in. In the last section an example problem is solved with the proposed algorithm, and some of the results are presented.

III. Crane as an Adaptive Structure

The linear excitation-response relation of any discrete structure, and as a special case that of a space truss, may be given as

$$\begin{bmatrix} \mathbf{0} & \mathbf{B} & \mathbf{0} \\ \mathbf{B}^T & \mathbf{0} & -\mathbf{I} \\ \mathbf{0} & -\mathbf{I} & \mathbf{K} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\xi} \\ \mathbf{s} \\ \mathbf{v} \end{Bmatrix} = \begin{Bmatrix} \mathbf{p} \\ \mathbf{v}_0 \\ \mathbf{o} \end{Bmatrix} \tag{1}$$

where the first row partition represents the nodal force equilibrium equations, the second row partition shows the geometric compatibility equations, and the last partition represents the constitutive relations of the material. For an N node, M member discrete structure with e number of degrees of freedom per node, a number of internal force component per member, b number of scalar deflection constraints due to supports, and f number of interelement force constraints, the response quantities are

$\boldsymbol{\xi}$: $(Ne-b)$-tuple of independent nodal deflections (2a)

$\mathbf{s}$: $(Ma-f)$-tuple of member forces (2b)

$\mathbf{v}$: $(Ma-f)$-tuple of induced member elongations (2c)

and the excitation quantities are

$\mathbf{p}$: $(Ne-b)$-tuple of external nodal forces (3a)

$\mathbf{v}_0$: $(Ma-f)$-tuple of prescribed member elongations (3b)

Note that for three-dimensional trusses, $e = 3$, $a = 1$, $f = 0$. If the three-dimensional structure is supported in a statically determinate fashion, $b = 6$.

Matrix $\mathbf{K}$ in Eq. (1) is a block diagonal matrix, where the mth diagonal block $\mathbf{K}^m$ relates the mth member's induced elongations $\mathbf{v}^m$ to its internal forces $\mathbf{s}^m$ as in $\mathbf{s}^m = \mathbf{K}^m\mathbf{v}^m$, i.e.,

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}^1 & & & \\ & \mathbf{K}^2 & & \\ & & \ddots & \\ & & & \mathbf{K}^M \end{bmatrix} \tag{4}$$

For an M member truss, $\mathbf{K} = \text{diag}[k^1, k^2, \ldots, k^M]$ with $k^m = (AE/L)^m$, $m = 1, 2, \ldots, M$, and A, L, and E are the cross-sectional area, the length, and the Young's modulus, respectively, of the member.

The matrix $\mathbf{B}$ is composed of the descriptions of the unit vectors in an inertially fixed reference coordinate system and the lengths of structural members. In the case of trusses, it consists of the descriptions of the bar unit vectors only. $\mathbf{B}$ needs to be updated when the orientations of the bars change. The structure is called statically determinate when the matrix $\mathbf{B}$ is a square matrix and nonsingular, i.e., when $Ne - b = Ma - f$ and $\det(\mathbf{B}) \neq 0$.

When a statically determinate discrete structure is a truss, the matrix $\mathbf{B}$ is of order M, and it is only $300e/M\%$ full. Its inverse may be obtained, column by column, and without generating $\mathbf{B}$ by a computational complexity of $\mathcal{O}(M^2)$ and a storage demand of $\mathcal{O}(M)$.[12]

In a statically determinate truss, from Eq. (1), one may obtain

$$\mathbf{s} = \mathbf{B}^{-1}\mathbf{p} \tag{5}$$

which shows that if $\mathbf{p} = \mathbf{o}$, then $\mathbf{s} = \mathbf{o}$, even if $\mathbf{v}_0 \neq \mathbf{o}$; i.e., no stresses develop in the determinate truss due to fabrication errors, thermal loads, or length changes caused by the control mechanisms (i.e., quantities represented by $\mathbf{v}_0$). Suppose $\mathbf{p} = \mathbf{o}$ and nonzero $\mathbf{v}_0$ is caused by the length-changing mechanisms. From Eq. (1), one may obtain the induced nodal deflections $\boldsymbol{\xi}$ as

$$\boldsymbol{\xi} = \mathbf{B}^{-T}\mathbf{v}_0 \tag{6}$$

which shows that for a given $\mathbf{v}_0$ there is a unique geometry change or vice-versa. Note that if only q number of components of $\mathbf{v}_0$ are made nonzero by the length-changing mechanisms, then we may be able to control any q number of components of $\boldsymbol{\xi}$, provided that the $q \times q$ submatrix formed by the intersection of the q columns of $\mathbf{B}^{-T}$ corresponding to the nonzero controls and the q rows of $\mathbf{B}^{-T}$ corresponding to the selected components of $\boldsymbol{\xi}$ is of rank q.

IV. Motion of Payload

The crane and all the masses in its workspace are actually on orbits around the world. Additional energy is required to move a mass farther away from the Earth, and negative energy is required to bring it close to the Earth. Also, additional energy is required to move the mass out of its orbital plane. However, for the purposes of this work we assume that zero

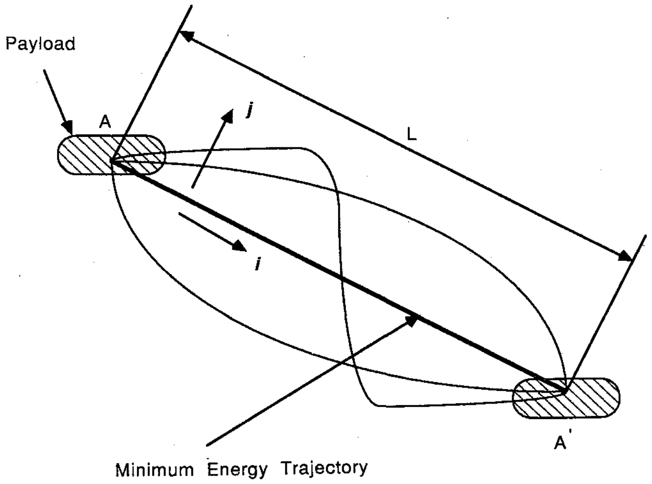

Fig. 2 Minimum-energy trajectory for end-effector (with zero-acceleration field in the workspace of the crane assumed).

net energy is required to move a mass m from one point to another in the workspace of crane (otherwise, we have to require the identification of the orbits of the payload before and after the crane's maneuver and indulge in many computations that are spurious to the objectives of this paper). So when a reference is made to payload trajectory, it merely refers to a curve segment in the workspace of the crane that has a zero-acceleration field.

The function of the space crane is to move the mass-center of a payload of mass m that is at rest with its mass-center at point A at time $t = 0$, to a point A' at time $t = T$ and leave it there with zero velocity and acceleration. It is assumed here that the forces applied to the payload by the end-effector of the crane all pass through the mass-center of the payload, thus, no rotational momentum is imparted to the payload. It is assumed that the rotational orientation of the payload needs no maneuvering, or if a rotational maneuver is needed, it will be done by the end-effector once the payload is at its destination. During the maneuver, we assume that no rotational momentum develops either at the mass-center of the payload, or at the joint of the truss where the end-effector is placed. Therefore, in the following discussions, the payload, its mass-center, the end-effector, and the joint of the space crane where the end-effector is placed all refer to the same point. Let L denote the distance between points A and A'. The quantities m, L, and T are the characteristic parameters of the maneuver to move the payload from point A to point A'.

In Fig. 2, various trajectories are shown for the maneuver. For all trajectories other than the straight-line AA', in addition to the axial momentum, we have to impart lateral momentums to the payload, first away from the line segment AA', and then toward the line segment AA', in order to place the payload at its target point A'. Unless there is a barrier, these lateral momentums are not warranted, and they unnecessarily increase the energy required by the maneuver. Therefore, we may state that the minimum-energy trajectory of the payload is the line segment AA'.

Let v_0 denote the maximum velocity the payload acquires during its trip to A' along the minimum-energy trajectory. The energy necessary for raising the payload's velocity from zero to v_0 is $(1/2)mv_0^2$. Since the payload is expected to be at rest at the destination point A', we have to spend the same amount of energy [i.e., $(1/2)mv_0^2$] to reduce the payload's velocity back to zero. Therefore, the total energy needed by the maneuver is

$$E = mv_0^2 \tag{7}$$

In addition to m, T, and L, the energy E is also a characteristic parameter of the maneuver.

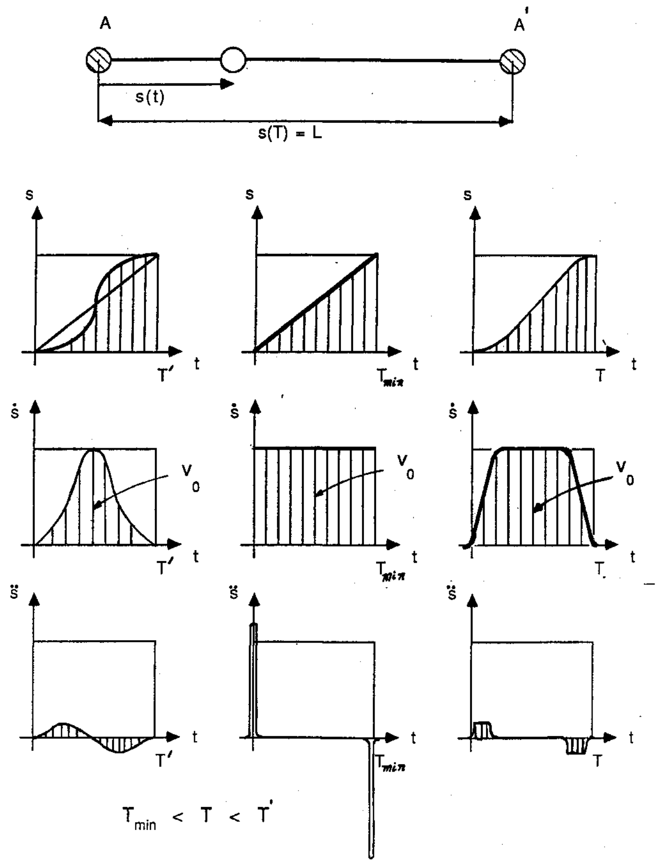

Fig. 3 Optimal moves along trajectory.

Given the energy E and payload mass m, from Eq. (7) one may compute the maximum payload velocity as

$$v_0 = \sqrt{E/m} \tag{8}$$

If this velocity can be maintained at every point of the trajectory AA', then the minimum maneuver completion time $T_{\min}$ may be computed as

$$T_{\min} = L/v_0 \tag{9}$$

To achieve $T_{\min}$, we have to impart to the payload at $t = 0$ a momentum mv_0 in the AA' direction, and at $t = T_{\min}$ a momentum of $-mv_0$ in the same direction. Since such jolts would cause both the crane and payload to vibrate, one may prefer the payload to reach and then lose its maximum speed v_0 not instantaneously but gradually. This will increase the maneuver completion time from $T_{\min}$ to T (see Fig. 3), but the undesired vibrations will be eliminated.

Let $\boldsymbol{i}$ denote the unit vector along the minimum energy trajectory AA' of the payload with mass m. Let $s(t)$ denote the distance of the payload from point A at time t. Given the maneuver parameters m, L, T, and E, we assume that $s(t)$ is already determined such that

$$s(0) = \dot{s}(0) = \ddot{s}(0) = 0, \quad s(T) = L, \quad \dot{s}(T) = \ddot{s}(T) = 0$$
$$\dot{s}_{\max} = \sqrt{E/m}, \quad \ddot{s}_{\max} = \alpha g \tag{10}$$

where g is the nominal gravitational acceleration of the Earth, and α is a sufficiently small prescribed constant. Then at any time t, the vector from A to the payload is $s(t)\boldsymbol{i}$, and the force acting on the crane is $-m\ddot{s}(t)\boldsymbol{i}$.

The payload trajectory defined by $s(t)\boldsymbol{i}$ will be used in the remainder of this study. However, any other trajectory, not necessarily optimal, may also be used.

V. Incremental Computation of Controls in Rigid Crane Corresponding to a Given Trajectory

Suppose that the crane is a determinate truss consisting of M rigid bars and q number of mechanisms placed in q of the M bars, controlling the length changes in these bars. Let the q-tuple $\mathbf{v}'^r(t)$ list the length changes in the q bars at time t (superscript r for rigid). The components of $\mathbf{v}'^r(t)$ are the values of the controls at time t. Let r denote the position vector of the attachment point (relative to an inertially fixed point Θ) of the crane with the payload. The motion of the attachment point is a function of the controls, i.e.,

$$r = r[\mathbf{v}'^r(t)] \tag{11}$$

Defining $r_A = r[\mathbf{v}'^r(0)]$ and $r_{A'} = r[\mathbf{v}'^r(T)]$ as position vectors of the terminal points of the trajectory AA', and using the optimal trajectory $s(t)i$ studied in the previous section, from Fig. 4, we may write

$$r[\mathbf{v}'^r(t)] = r[\mathbf{v}'^r(0)] + s(t)i; \qquad 0 \le t \le T \tag{12}$$

In this highly nonlinear (with respect to the unknowns listed in $[\mathbf{v}'^r(t)]$) vectorial equation, the right-hand side is known at all times, and we are expected to solve for the controls $[\mathbf{v}'^r(t)]$. We will show that we can fulfill this expectation without even generating the explicit expressions for the components of the nonlinear vector equation.

Let Δt denote a small time increment and define $t' = t - \Delta t$. Let $\mathbf{r}$ and $\mathbf{i}$ denote the descriptions of r and i, respectively, in the inertially fixed reference frame. With this notation, we may rewrite Eq. (12) as

$$\mathbf{r}[\mathbf{v}'^r(t' + \Delta t)] = \mathbf{r}[\mathbf{v}'^r(0)] + s(t)\mathbf{i} \tag{13}$$

or by Taylor series expansion

$$\mathbf{r}[\mathbf{v}'^r(t')] + [\mathbf{r},_{\mathbf{v}'^r}]_{t'}\Delta\mathbf{v}'^r + \mathbf{e} = \mathbf{r}[\mathbf{v}'^r(0)] + s(t)\mathbf{i} \tag{14}$$

where

$$\Delta\mathbf{v}'^r = \mathbf{v}'^r(t) - \mathbf{v}'^r(t') \tag{15}$$

$\mathbf{e}$ is the remainder of the expansion, and $[\mathbf{r},_{\mathbf{v}'^r}]_{t'}$ is the Jacobian matrix of $\mathbf{r}$ with respect to the components of $\mathbf{v}'^r$, evaluated at time t'. By rewriting Eq. (12) for time t', and subtracting from Eq. (14) one may obtain

$$[\mathbf{r},_{\mathbf{v}'^r}]_{t'}\Delta\mathbf{v}'^r + \mathbf{e} = [s(t) - s(t')]\mathbf{i} \tag{16}$$

For sufficiently small Δt, the remainder term may be ignored

$$[\mathbf{r},_{\mathbf{v}'^r}]_{t'}\Delta\mathbf{v}'^r = \Delta s\mathbf{i} \tag{17}$$

where

$$\Delta s = s(t) - s(t') \tag{18}$$

This equation relates the small changes in the lengths of length-controlled bars of the truss with the displacements of the node where the end-effector is placed.

In general, we may like to control various displacement components of various nodes for the proper maneuvering of the crane. Suppose we like to control p number of nodal displacement components. From Eq. (6), by deleting all rows but those of the selected p number of displacement components, one may write

$$\xi_p' = (\mathbf{B}^{-1})_p^T\mathbf{v} \tag{19}$$

where ξ_p' lists the selected p number of displacement components, $(\mathbf{B}^{-1})_p^T$ represents the p rows of $\mathbf{B}^{-T}$ (or equivalently, p columns of $\mathbf{B}^{-1}$) corresponding to the selected p number of displacement components, and $\mathbf{v}$ lists the elongation of all bars. Let $\mathbf{C}^{q,p}$ denote the matrix $(\mathbf{B}^{-1})_p^T$ after its columns associated with noncontrolled bars are removed. Note that $\mathbf{C}^{q,p}$ is a $p \times q$ matrix, and it relates the length changes in the q number of controlled bars with the p number of selected displacement components. (Note that in the simple case we are treating, the selected displacement components are those of the node where the end-effector is placed, and $p = e$). Since the matrix $[\mathbf{r},_{\mathbf{v}'^r}]_{t'}$ in Eq. (17) does exactly the same job, we conclude that

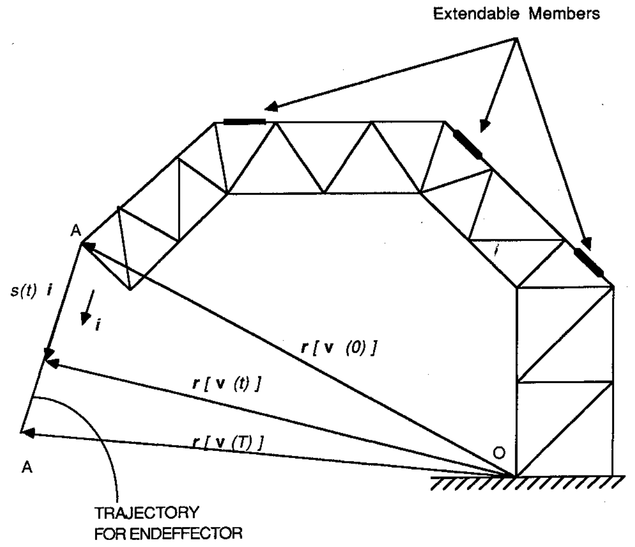

Fig. 4 Relationship between position vector *r(t)* and *r*(0).

$$[\mathbf{r},_{\mathbf{v}'^r}]_{t'} = \mathbf{C}^{q,p} \tag{20}$$

Using Eq. (20) and defining

$$\Delta\xi'^r = \Delta s\mathbf{i} \tag{21}$$

we may rewrite Eq. (17) as

$$\mathbf{C}^{q,p}\Delta\mathbf{v}'^r = \Delta\xi'^r \tag{22}$$

When $p > e$, the right-hand side of Eq. (22) will be a known p-tuple computable from the known trajectory of the payload.

When $p = q$ and $\det(\mathbf{C}^{q,p}) \neq 0$, we may be able to find the numerical values of the incremental controls at time t' that would bring the node on the trajectory from point $\mathbf{r}[\mathbf{v}'^r(0)] + s(t')\mathbf{i}$ to point $\mathbf{r}[\mathbf{v}'^r(0)] + s(t)\mathbf{i}$ as

$$\Delta\mathbf{v}'^r = \mathbf{C}^{q,p^{-1}}\Delta\xi'^r \tag{23}$$

However, when the rank of $\mathbf{C}^{q,p}$ is less than p, then the crane is not capable of displacing itself according to the prescribed trajectory. When the rank of $\mathbf{C}^{q,p}$ is p and $q > p$, there may be many possibilities for the maneuver. We may activate any p of the q control mechanisms. However, for a robust control, we may activate only those p number of mechanisms that are associated with the largest magnitude pth-order minor of $\mathbf{C}^{q,p}$.

Suppose we select S number of equally spaced stations $k, k = 0,1,2, \ldots, S$ in the maneuvering time interval $[0,T]$ such that the time t_k at station k may be expressed as

$$t_k = k\Delta t \tag{24}$$

where

$$\Delta t = T/S \tag{25}$$

Using the notation of

$$h(t_k) = h_k \tag{26}$$

for the case of $p = q = e$ and $\det([\mathbf{C}^{q,p}_{k-1}]) \neq 0$, we may rewrite Eqs. (15) and (23) as

$$\left.\begin{aligned} \mathbf{v}^r_k &= \mathbf{v}^r_{k-1} + \Delta\mathbf{v}^r_{k-1} \\ \Delta\mathbf{v}^r_{k-1} &= [\mathbf{C}^{q,p}_{k-1}]^{-1}\Delta\xi'^r \end{aligned}\right\} \quad k = 1,2, \ldots, S \tag{27}$$

where $\mathbf{v}^r_0 = 0$ and $\mathbf{C}^{q,p}$ is obtainable from $\mathbf{B}^{-T}_{k-1}$ by deleting the columns related with uncontrolled bars and the rows related with the uncontrolled displacement components.

Since $\mathbf{B}_{k-1}$ depends on the description of unit vectors of the bars at time station $k-1$, we need to update the coordinates of all the nodes at each step. This means that not only the columns corresponding to the prescribed displacement components but also all of the other displacement components need to be computed. Let $(Ne-b)$-tuple $\Delta\xi^r_{k-1}$ denote the complete list of incremental nodal displacements caused by the controls $\Delta\mathbf{v}'^r_{k-1}$. From Eq. (6) we may write

$$\Delta\xi^r_{k-1} = [\mathbf{B}^{-T}_{k-1}]''\Delta\mathbf{v}'^r_{k-1} \tag{28}$$

where $[\mathbf{B}^{-T}_{k-1}]''$ is the matrix $\mathbf{B}^{-T}_{k-1}$ after the columns related with the noncontrol bars are removed.

Clearly, the computational success of Eqs. (27) and (28) depends on the computational complexity of obtaining $\mathbf{B}^{-1}$ from $\mathbf{B}$. Luckily, for determinate trusses this can be done with $\mathcal{O}(M^2)$ computational complexity with a storage demand of $\mathcal{O}(M)$. For the algorithm, see Ref. 12.

VI. Compensation of Incremental Controls for Elastic Deformations

The incremental controls given in the previous section assume that the crane does not deform under forces. In reality, as the crane imparts momentum to the payload, the crane itself is subjected to a force in the opposite direction of the momentum vector. Let f denote this force. Since the motion of the payload is described by $s(t)i$, we may write

$$f = -\frac{\mathrm{d}^2 s}{\mathrm{d}t^2}mi \tag{29}$$

or by denoting the description of f in the inertially fixed reference system by $\mathbf{f}$

$$\mathbf{f} = -\frac{\mathrm{d}^2 s}{\mathrm{d}t^2}m\mathbf{i} \tag{30}$$

Under the effect of force f, the crane will deform, causing the end-effector to depart from the prescribed trajectory. However, with the knowledge of f the deformation can be predicted and the controls adjusted in order to keep the end-effector on the prescribed trajectory.

Let $\Delta\mathbf{f}_{k-1}$ denote the change in the payload-induced force acting on the crane as the payload moves from its position at time t_{k-1} to its position at time t_k. Using Eq. (30) and the notation of Eq. (26), one may write

$$\Delta\mathbf{f}_{k-1} = -\left[\left(\frac{\mathrm{d}^2 s}{\mathrm{d}t^2}\right)_k - \left(\frac{\mathrm{d}^2 s}{\mathrm{d}t^2}\right)_{k-1}\right]m\mathbf{i} \tag{31}$$

Since the motion of the payload is known, $\Delta\mathbf{f}_{k-1}$ is a known force acting on the crane at the node where the end-effector is located. From Eq. (1), with $\mathbf{v}_0 = 0$, $\mathbf{B} = \mathbf{B}_{k-1}$, the induced elastic incremental nodal displacements $\Delta\xi^{e1}_{k-1}$ may be computed as

$$\Delta\xi^{e1}_{k-1} = \mathbf{B}^{-T}_{k-1}\mathbf{K}^{-1}[\mathbf{B}^{-1}_{k-1}]''\Delta\mathbf{f}_{k-1} \tag{32}$$

where $\mathbf{K}$ is as defined in Eq. (4). Note that the right-hand side of Eq. (32) should be computed from right to left (superscript e stands for elastic).

Let $\Delta\xi'^{e1}_{k-1}$ denote the p-tuple after the entries of $\Delta\xi^{e1}_{k-1}$ not related with the end-effector joint(s) are removed. Let $\Delta\mathbf{v}'^{e}_{k-1}$ denote the incremental controls to nullify $\Delta\xi'^{e1}_{k-1}$, i.e., the deviations from the prescribed trajectory of the end-effector support. From the prescribed displacement vs control relations given in Eq. (22), one may write

$$\mathbf{C}^{q,p}\Delta\mathbf{v}'^{e}_{k-1} = -\Delta\xi'^{e1}_{k-1} \tag{33}$$

from where $\Delta\mathbf{v}'^{e}_{k-1}$ may be solved as discussed in the previous section.

The complete list of incremental independent nodal displacements $\Delta\xi^{e2}_{k-1}$ may be obtained from the controls vs nodal displacements relation (28) as

$$\Delta\xi^{e2}_{k-1} = [\mathbf{B}^{-T}_{k-1}]''\Delta\mathbf{v}'^{e}_{k-1} \tag{34}$$

Let $\Delta\xi^{e}_{k-1}$ denote the complete list of incremental independent nodal displacements due to incremental elastic deformations caused by $\Delta\mathbf{f}_{k-1}$ and the incremental controls $\Delta\mathbf{v}'^{e}_{k-1}$. Then

$$\Delta\xi^{e}_{k-1} = \Delta\xi^{e1}_{k-1} + \Delta\xi^{e2}_{k-1} \tag{35}$$

by superposition, which is permissible as a result of the linear behavior of the crane between time stations $k-1$ and k.

VII. Compensation of Incremental Controls for the Support Movements

The space crane is expected to be attached to a space platform or a space construction facility, both of which are of finite mass. When the crane imparts a momentum to the payload, the crane and supporting structure are subjected negative of that momentum. Depending on the mass of the payload relative to the sum of the masses of the crane and its supporting structure, the supports of the crane will move. If this movement can be prevented completely by the control system of the supporting structure, then no additional compensation in the crane's control is necessary. However, if the movements of the supporting structure are not prevented, or partially prevented, then an additional compensation in the crane's control is necessary.

Here we assume that the supporting structure is much stiffer than the crane, and it moves as a rigid body some known amount. Let $\Delta\xi^{s1}_{k-1}$ denote the incremental independent displacements of the nodes at time station $k-1$ due to rigid-body movement of the supporting structure between time stations $k-1$ and k (superscript s stands for support movements). Let $\Delta\xi'^{s1}_{k-1}$ denote the p-tuple after the entries of $\Delta\xi^{s1}_{k-1}$ not related with the end-effector joint(s) are removed. Let $\Delta\mathbf{v}'^{s}_{k-1}$ denote the incremental controls to nullify $\Delta\xi'^{s1}_{k-1}$, which are the deviations from the prescribed trajectory of the end-effector support due to rigid-body movement of the crane. From the prescribed displacement vs control relations given in Eq. (22), one may write

$$\mathbf{C}^{q,p}\Delta\mathbf{v}'^{s}_{k-1} = -\Delta\xi'^{s1}_{k-1} \tag{36}$$

from where $\Delta\mathbf{v}'^{s}_{k-1}$ may be solved as discussed earlier. The complete list of incremental independent nodal displacements $\Delta\xi^{s2}_{k-1}$ caused by the controls $\Delta\mathbf{v}'^{s}_{k-1}$ may be obtained from the controls vs nodal displacements relation (28) as

$$\Delta\xi^{s2}_{k-1} = [\mathbf{B}^{-T}_{k-1}]''\Delta\mathbf{v}'^{s}_{k-1} \tag{37}$$

Let $\Delta\xi^{s}_{k-1}$ denote the complete list of incremental independent nodal displacements caused by the rigid-body movement of the crane and the incremental controls $\Delta\mathbf{v}'^{s}_{k-1}$. Then

$$\Delta\xi^{s}_{k-1} = \Delta\xi^{s1}_{k-1} + \Delta\xi^{s2}_{k-1} \tag{38}$$

by superposition, since the crane behaves linearly between time stations $k-1$ and k.

VIII. Computational Complexity of an Incremental Step

Suppose the cumulative nodal displacements ξ_{k-1} and the cumulative controls $\mathbf{v}'_{k-1}$ at time station $k-1$ are known. According to the foregoing discussions, we may obtain the cumulative nodal displacements ξ_k and the cumulative controls $\mathbf{v}'_k$ as follows:

$$\mathbf{v}'_k = \mathbf{v}'_{k-1} + \Delta\mathbf{v}'_{k-1} \tag{39}$$

$$\xi_k = \xi_{k-1} + \Delta\xi_{k-1} \tag{40}$$

where

$$\Delta\mathbf{v}'_{k-1} = [\mathbf{C}^{p,p}_{k-1}]^{-1}(\Delta\xi'^{r}_{k-1} - \Delta\xi'^{e1}_{k-1} - \Delta\xi'^{e1}_{k-1}) \tag{41}$$

$$\Delta\xi_{k-1} = [\mathbf{B}^{-T}_{k-1}]''\Delta\mathbf{v}'_{k-1} + \mathbf{B}^{-T}_{k-1}\mathbf{K}^{-1}[\mathbf{B}^{-1}_{k-1}]''\Delta\mathbf{f}_{k-1} + \Delta\xi^{s1}_{k-1} \tag{42}$$

where all other symbols are as defined previously. The computational complexity of Eq. (41) is basically $\mathcal{O}(p^3)$, and the computational complexity of Eq. (42) is basically $\mathcal{O}(M^2)$.[12] Since $p \ll M$, the computational complexity of the computations of ξ_k and $\mathbf{v}_k$ by Eqs. (39) and (40) is essentially $\mathcal{O}(M^2)$.

It is interesting to note what the results will be if one used the finite element method as a basis for the computation of crane controls instead of the special $\mathcal{O}(M^2)$ algorithm in Ref. 12. Consider Eq. (1) rewritten as

$$\mathbf{Bs} = \mathbf{p} \tag{43}$$

$$\mathbf{B}^T\xi - \mathbf{v}_0 = \mathbf{v} \tag{44}$$

and

$$\mathbf{Kv} = \mathbf{s} \tag{45}$$

In the finite element method, Eq. (45) for $\mathbf{s}$ is substituted in Eq. (43) and the resultant equation for $\mathbf{v}$ is substituted in Eq. (44) to obtain

$$\mathbf{BKB}^T\xi = \mathbf{BKv}_0 + \mathbf{p} \tag{46}$$

from which one obtains ξ as

$$\xi = (\mathbf{BKB}^T)^{-1}(\mathbf{BKv}_0 + \mathbf{p}) \tag{47}$$

The matrix $(\mathbf{BKB}^T)$ is the global stiffness matrix of the truss structure. If det $\mathbf{B} \neq 0$ and $\mathbf{p} = 0$, then one can also write

$$\xi = \mathbf{B}^{-T}\mathbf{K}^{-1}\mathbf{B}^{-1}\mathbf{BKv}_0 \tag{48}$$

and thus obtain

$$\xi = \mathbf{B}^{-T}\mathbf{v}_0 \tag{49}$$

which is the same as Eq. (6). Therefore, if we were to use the finite element method, we first have to assemble the global stiffness matrix $\mathbf{BKB}^T$ and also the nodal loads given by $\mathbf{BKv}_0$. Then we invoke Eq. (46) to obtain the nodal displacements. The assembly of the global stiffness matrix and its inversion will cost $\mathcal{O}(M^2w^2)$,[11] where w is the half-bandwidth of the finite element model of the truss. Therefore, the speed of the

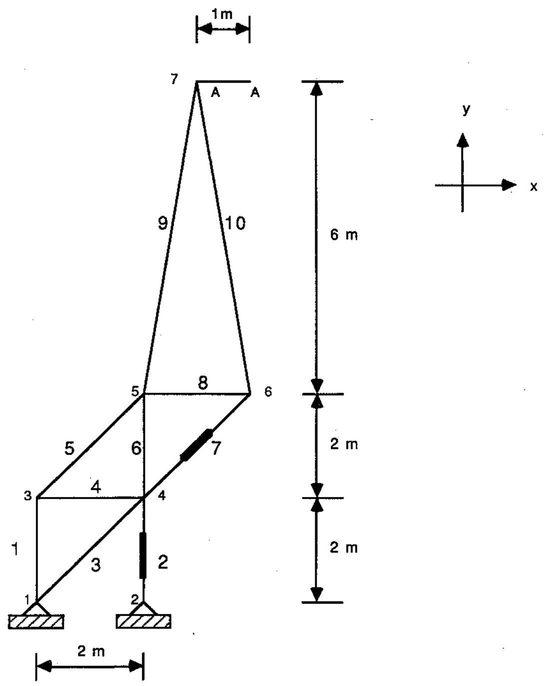

Fig. 5 Crane of example problem.

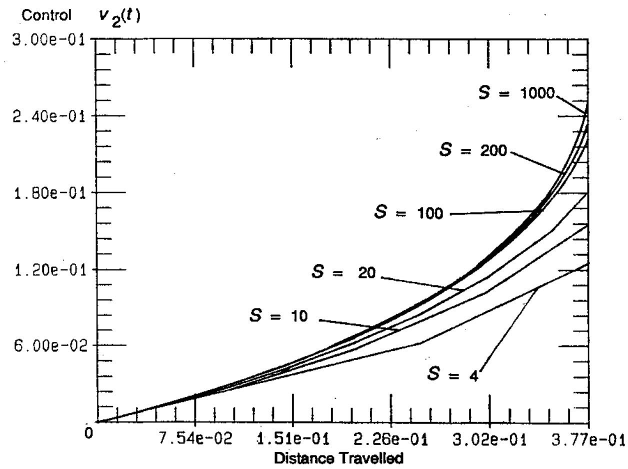

Fig. 6a Control in bar 2 as a function of distance traveled along the trajectory and number of incremental steps S used in computations.

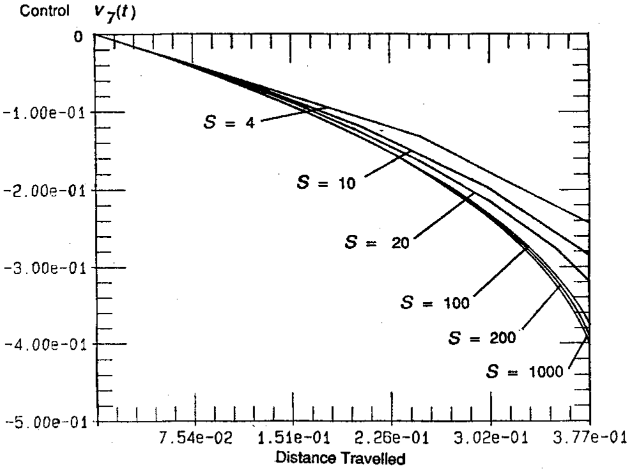

Fig. 6b Control in bar 7 as a function of distance traveled along the trajectory and number of incremental steps S used in computations.

finite element method based control algorithm will be dependent on w and the assembly process. The assembly process assumes a significant role here since the crane is an adaptive structure and for every time station the global stiffness matrix needs to be reassembled. This is in contrast to the algorithm cited in this paper, which solves Eq. (6) and therefore involves no assembly process. Not only is the computational cost higher if the finite element method is used, but also the accuracy of the control deteriorates due to round-off error. This is because Eq. (47) involves more calculations (and hence it is more prone to round-off) and also the condition of the equation can be deteriorated by a bad stiffness distribution. For determinate structures, stiffness information is not needed in computing the slow motion control under rigid assumptions, and hence it is more appropriate to use Eq. (6) in computing the control.

IX. Example Problem

As an example problem to illustrate the control algorithm, consider the crane shown in Fig. 5. It is assumed that node 7 has the end-effector attached to it and bars 2 and 7 are length-adjustable. An optimal (straight-line) trajectory of length 1.0 m beginning at point A and ending at A' is shown in Fig. 5. The trajectory is chosen so that point A' is beyond the reach of the crane, i.e., lies outside the workspace of the crane. The unknown control elongations in bars 2 and 7 are to be determined so that node 7 moves along the prescribed trajectory.

The incremental controls are computed as in Eq. (41) where only the rigid contribution is considered and the contributions due to elasticity and support movement are ignored for convenience. The increments are accumulated to compute the cumulative control at any instant. Figures 6a and 6b show the cumulative controls in bars 2 and 7, respectively, for different increments in the distance along the trajectory Δs. The smaller the value of Δs, the more accurate the incremental control computation. This is easily seen in Figs. 6a and 6b where the controls $v_2(t)$ and $v_7(t)$ converge to nonlinear curves as Δs becomes smaller. The controls are plotted only up to $s(t) \approx 0.38$ m since the Jacobian vanishes at that point, indicating that the crane cannot be taken any further along this trajectory by this set of controls. The curves in Figs. 6a and 6b signify that bar 2 should be elongated and bar 7 should be shortened, which concurs with our physical intuition.

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